Panel Experiments and Dynamic Causal Effects: A Finite Population Perspective

Iavor Bojinov Ashesh Rambachan Neil Shephard

ASSA 2021
Advances in Panel Data Methods

Panel Experiments

- Panel experiments: sequentially assign units to random intervention, measure response and repeat procedure over a fixed period time.

- Panel experiments are widely used in biostatistics, epidemiology and psychology (e.g. Robins, 1986, 1994; Murphy et al., 2001; Murphy, 2003; Lillie et al., 2011)).

- Example: Patients receive antiretroviral treatment for HIV over many months (Hernan and Robins, 2019).
  - Some patients may have received treatment in all months
    Others may have never received treatment
    Others receive treatment in some months but not others.

  What is the both the contemporaneous and dynamic causal effect of antiretroviral treatment on health outcomes?

- Panel experiments useful because (1) potential gains in power (Bellemare et al., 2014, 2016); (2) uncover treatment heterogeneity across units and time (Czibor et al., 2019).

Panel Experiments

- Panel experiments: sequentially assign units to random intervention, measure response and repeat procedure over a fixed period time.
- Despite benefits, panel experiments are rare in economics.
- Key concern focuses on how dynamic treatment effects may induce biases in conventional experimental estimators (Charness et al., 2012).
    - If treatment yesterday affects outcomes today, how should this be accounted for in analysis stage?
- **This Paper**: Tackle by developing finite population framework for analyzing panel experiments.
  $\hookrightarrow$ Enable applied researchers to use panel experiments to answer interesting economic questions.

Overview of Paper

- Develop potential outcome panel model and define useful class of dynamic causal estimands.
  - How does changing assignment affect outcomes after p periods?

- For this class of estimands, we provide nonparametric estimators that are
  1. unbiased over randomization distribution,
  2. asymptotically normally distributed as either number of experimental units or sample periods grows large (finite pop. CLT).

  Provide methods for inference on both weak and sharp null hypotheses.

- We analyze limiting bias of standard linear estimators commonly employed on panel data.
  - Conventional estimators are biased whenever there exists (1) dynamic causal effects, (2) serial correlation in assignments.

- Illustrate our methods by re-analyzing a panel experiment on rational cooperation conducted by Andreoni and Samuelson (2006).

Outline

Potential outcome panel and dynamic causal effects

Nonparametric estimation and testing

Estimation in a linear potential outcome panel

Empirical application: rational cooperation in games

Assignment Panels and Potential Outcomes

- Balanced panel setting in which N units observed over T periods.
- Treatment $W_{i,t}$ is assigned to each unit i in period t.
  Treatment has finite support, and is typically binary.
- Assignment Panel: matrix of treatments assigned to all units over sample period, $W_{1:N,1:T}$

$$\begin{pmatrix} W_{1,1} & \dots & W_{1,t} & \dots & W_{1,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{i,1} & \dots & W_{i,t} & \dots & W_{i,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{N,1} & \dots & W_{N,t} & \dots & W_{N,T} \end{pmatrix}$$

Assignment Panels and Potential Outcomes

- Balanced panel setting in which N units observed over T periods.
- Treatment $W_{i,t}$ is assigned to each unit i in period t. Treatment has finite support, and is typically binary.
- Assignment path: sequence of treatments assigned to unit i over sample period, $W_{i,1:T}$

$$\begin{pmatrix} W_{1,1} & \dots & W_{1,t} & \dots & W_{1,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{i,1} & \dots & W_{i,t} & \dots & W_{i,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{N,1} & \dots & W_{N,t} & \dots & W_{N,T} \end{pmatrix}$$

Assignment Panels and Potential Outcomes

- Balanced panel setting in which N units observed over T periods.
- Treatment $W_{i,t}$ is assigned to each unit i in period t. Treatment has finite support, and is typically binary.
- Cross-sectional assignment: treatments assigned to all units at period t, $W_{1:N,t}$.

$$\begin{pmatrix} W_{1,1} & \dots & W_{1,t} & \dots & W_{1,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{i,1} & \dots & W_{i,t} & \dots & W_{i,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ W_{N,1} & \dots & W_{N,t} & \dots & W_{N,T} \end{pmatrix}$$

Assignment Panels and Potential Outcomes

- Balanced panel setting in which N units observed over T periods.

- Treatment $W_{i,t}$ is assigned to each unit i in period t. Treatment has finite support, and is typically binary.

- Potential outcome $Y_{i,t}(w_{1:N,1:T})$: Outcome that would be observed for unit i at period t along assignment panel.
  - In principle allows for arbitrary spillovers across units and time periods

- Potential outcomes as a function of assignment paths first appears in Robins (1986); further developed extensively in biostatistics.
  - Our work differs by avoiding super-population/random sampling arguments.
    All arguments conditioned on the potential outcomes and uncertainty arises from randomness in assignment panel.

Assignment Panels and Potential Outcomes

- Balanced panel setting in which N units observed over T periods.
- Treatment $W_{i,t}$ is assigned to each unit i in period t. Treatment has finite support, and is typically binary.
- Potential outcome $Y_{i,t}(w_{1:N,1:T})$: Outcome that would be observed for unit i at period t along assignment panel.
  - In principle allows for arbitrary spillovers across units and time periods
- Unified generalization design-based framework for analysis of cross-sectional experiments (Imbens and Rubin, 2015) and time series experiments (Bojinov and Shephard, 2019).
  - $T = 1 \leftrightarrow$ cross-sectional experiment.
  - $N = 1 \leftrightarrow$ time series experiment.

Potential outcome panel

- Introduce restrictions on potential outcomes that limit spillovers across units and time periods.

- Non-interference: No spillovers across units.
  - Potential outcome for unit *i* at time *t* does not depend on assignment paths of other units.

- Non-anticipation: Potential outcome for unit *i* at time *t* does not depend on future assignments.
  - For paths $w_{i,1:T}, \tilde{w}_{i,1:T}$, $Y_{i,t}(w_{i,1:T}) = Y_{i,t}(\tilde{w}_{i,1:T})$ whenever $w_{i,1:t} = \tilde{w}_{i,1:t}$.

- $\Longrightarrow$ Potential outcome for unit *i* at time *t* only depends on unit *i*'s assignment path up to time *t*, denoted $Y_{i,t}(w_{i,1:t})$.

Dynamic causal effects

- Dynamic causal effects compare potential outcomes for unit-i at time-t along different assignment paths
$$\tau_{i,t}(w_{i,1:t}, \tilde{w}_{i,1:t}) := Y_{i,t}(w_{i,1:t}) - Y_{i,t}(\tilde{w}_{i,1:t}).$$

- Our main focus is on lag-p dynamic causal effects
$$\tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}})(p) := \tau_{i,t}(\{w^{obs}_{i,1:t-p-1}, \mathbf{w}\}, \{w^{obs}_{i,1:t-p-1}, \tilde{\mathbf{w}}\}).$$
Measures effect of changing assignment path from $\tilde{\mathbf{w}}$ to $\mathbf{w}$ over periods $t-p$ to t, along observed assignment path up to $t-p-1$.

- By restricting paths $\mathbf{w}$ and $\tilde{\mathbf{w}}$ to share common features and averaging over possible paths, we may obtain interesting dynamic causal effects.

Weighted average dynamic causal effects

- Weighted average dynamic causal effect summarizes average causal effect of switching assignment at period $t-p$, averaging over all possible paths from $t-p+1$ to t

$$\tau^{\dagger}_{i,t}(w,\tilde{w})(p) := \sum_{\mathbf{v}\in\mathcal{W}^p} a_{\mathbf{v}} \left\{ Y_{i,t}(w^{obs}_{i,1:t-p-1}, w, \mathbf{v}) - Y_{i,t}(w^{obs}_{i,1:t-p-1}, \tilde{w}, \mathbf{v}) \right\}.$$

  If treatment switched from $\tilde{w}$ to w at time $t-p$, what is the average effect on outcomes at time t?

- Weighted average dynamic causal effects are finite-population causal analogues of impulse response functions.
  - Rambachan and Shephard (2020): generalized impulse response function (Koop et al., 1996) equivalent to weighted average dynamic causal effect for particular choice of weights.

Averaging dynamic causal effects

- Target estimand: averages across time and units of lag-p dynamic causal effects.
  1. Time-t average:
  $$\bar{\tau}_t(\mathbf{w}, \tilde{\mathbf{w}})(p) := \frac{1}{N} \sum_{i=1}^{N} \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}})(p) \quad \leftarrow \text{ Heterogeneity across time}$$
  2. Unit-i average:
  $$\bar{\tau}_i(\mathbf{w}, \tilde{\mathbf{w}})(p) := \frac{1}{T-p} \sum_{t=p+1}^{T} \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}})(p) \quad \leftarrow \text{ Heterogeneity across units}$$
  3. Total average:
  $$\bar{\tau}(\mathbf{w}, \tilde{\mathbf{w}})(p) := \frac{1}{N(T-p)} \sum_{t=p+1}^{T} \sum_{i=1}^{N} \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}})(p).$$
- Def'ns extend to weighted average dynamic causal effects.

Assignment Mechanism

- Assignment mechanism is assumed to be known and sequentially randomized.
- Sequentially randomized: Cross-sectional assignment at time t only depends on past assignments and observed outcomes, not future nor unobserved past potential outcomes.
- Leading special case further assumes assignment mechanism is individualistic.
  - Conditional on own past assignments and outcomes, assignment for unit i independent of past assignments and outcomes of all other units.

Nonparametric estimation and testing: overview

- **So far**: defined a rich class of dynamic causal estimands.
  $\hookrightarrow$ Can we construct estimators for them? Testing and confidence intervals?

- **Next**: develop nonparametric Horvitz-Thompson type estimator of lag-p dynamic causal effect and develop two methods for inference.

- Conservative tests for weak null hypotheses based on finite population central limit theorem.
  - Requires assignment mechanism to be individualistic and satisfy an overlap/positivity condition.

- Exact tests for sharp null hypotheses based on randomization distribution (see Section 3.4 for details).
  - Requires assignment mechanism to be sequentially randomized.

- **In the paper**: develop analogous results for HT estimator of weighted average dynamic causal effects.

Propensity score and Horvitz-Thompson estimator

- Propensity score: conditional probability of assignment path given past treatments and outcomes

$$p_{i,t-p}(\mathbf{w}) := \Pr(W_{i,t-p:t} = \mathbf{w} | W_{i,1:t-p-1}, Y_{i,1:t}(W_{i,1:t-p-1}, \mathbf{w})).$$

- Probabilistic assignment mechanism: exists $C^L, C^U \in (0,1)$ such that

$$C^L < p_{i,t-p}(\mathbf{w}) < C^U.$$

- HT estimator of lag-p dynamic causal effect is

$$\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) := \frac{Y_{i,t}(w^{obs}_{i,1:t-p-1}, \mathbf{w})\mathbb{1}(w^{obs}_{i,t-p:t} = \mathbf{w})}{p_{i,t-p}(\mathbf{w})} - \frac{Y_{i,t}(w^{obs}_{i,1:t-p-1}, \tilde{\mathbf{w}})\mathbb{1}(w^{obs}_{i,t-p:t} = \tilde{\mathbf{w}})}{p_{i,t-p}(\tilde{\mathbf{w}})},$$

which is computable along observed treatment path.

Horvitz-Thompson Estimator

- **Theorem 3.1**: Under individualistic $+$ probabilitistic assignment, HT estimator is unbiased for lag-p dynamic causal effect over randomization distribution and can derive its variance. details

- Error in estimating lag-p causal effect w/ HT estimator is a martingale difference sequence through time and conditionally independent across units.

- Variance of HT estimator is not identified (depends on unobserved potential outcomes).
  But we construct an unbiased estimator for an upper-bound (Lemma 3.2). details

Finite population CLT and conservative inference

- Martingale difference properties enable us to develop finite pop. CLTs to appropriately scaled and centered versions of averages of HT estimator.

- **Theorem 3.2**: Under individualistic, probabilistic assignment $+$ bounded potential outcomes,
  1. Scaled $+$ centered HT estimator of time-t, avg. dynamic causal effect $\xrightarrow{d} N(0, 1)$ as $N \to \infty$.
  2. Scaled $+$ centered HT estimator of unit-i, avg. dynamic causal effect $\xrightarrow{d} N(0, 1)$ as $T \to \infty$.
  3. Scaled $+$ centered HT estimator of total avg. dynamic causal effect $\xrightarrow{d} N(0, 1)$ as $NT \to \infty$.

  details

- Construct conservative confidence intervals and test weak nulls that average dynamic causal effects are zero using finite population CLTs and estimators of variance bound.

Linear potential outcome panel

- Much existing research on causal inference in panel data focuses on estimating dynamic causal effects using linear models (e.g., linear regression w/ unit/time fixed effects).

- We analyze estimand identified by such estimators in *linear potential outcome panel*.

- Linear potential outcome panel assumes potential outcome satisfies

$$Y_{i,t}(w_{i,1:t}) = \beta_{i,t,0} w_{i,t} + \ldots + \beta_{i,t,t-1} w_{i,1} + \epsilon_{i,t} \quad \forall t \geq 1,$$

  Coeff. $\beta_{i,t,j}$ are dynamic causal coefficients.

  - Leading example arises from autoregressive potential outcome model details

Estimation with linear models: unit fixed effects

- Characterize finite pop. prob. limit of unit fixed effects estimator in linear potential outcomes model, allowing for arbitrary heterogeneity in causal coefficients.

- **Proposition 4.2**: Prob. limit of unit fixed effects estimator as $N \to \infty$ decomposed into three terms:
    1. Weighted avg of *contemporaneous* dynamic causal coefficients w/ weights proportional to unit-specific variance of treatment assignments.
    2. Weighted avg of *lagged* dynamic causal coefficients w/ weights proportional to unit-specific autocovariance of treatment assignments.
    3. Additional error due to possible relationship b/w average treatment and "null" assignment path across units.

  details example

- **In the paper**: provide analogous results for two-way fixed effects estimator (Proposition 4.3).

Andreoni & Samuelson (2006): rational cooperation in games

- Andreoni & Samuelson (2006): Develop game-theoretic model of “rational cooperation” in two-period prisoners’ dilemma.
  - Higher payoffs are in period two $\Longrightarrow$ more players will cooperate in period one.

- Design of panel experiment: details
  - Subjects play 20 games of two-period prisoners’ dilemma.
  - Distribution of payoffs across two periods randomly assigned.
  - Subjects randomly matched into pairs for each game.

  In total, $N = 110$ participants over $T = 20$ games.

- Subjects may learn about structure of stage game w/ each play, so researchers may worry about dynamic treatment effects.

- Our Goals:
  1. investigate whether there appear to be dynamic treatment effects,
  2. test AS’ prediction in manner that is robust to possible dynamic treatment effects.

Rational cooperation in prisoners' dilemma

- Outcome $Y_{i,t}$: whether participant *i* cooperated in period 1 of game *t*.
- Treatment $W_{i,t}$: whether participant *i* played game *t* in which payoffs more concentrated in period 2.
- Estimand: Total lag-p weighted avg dynamic causal effect $\bar{\tau}^{\dagger}(1, 0; p)$.
- Summarizes dynamic causal effect of treatment on outcome, averaged across all units and time periods.
    - $p = 0$: causal effect of higher payoffs in period 2 of *current* game on cooperation in *current* game $\implies$ AS' prediction?
    - $p > 0$: causal effect of higher payoffs in period 2 of *previous* game on cooperation in *current* game $\implies$ dynamic causal effects?

Rational cooperation in prisoners' dilemma

rand. dists

	lag-p			
	0	1	2	3
Point estimate, $\hat{\bar{\tau}}^{\dagger}(1,0)(p)$	0.285	0.058	0.134	0.089
Randomization p-value	0.000	0.263	0.012	0.114

- Strongly reject sharp null of no contemporaneous dynamic causal effects for all units $\implies$ confirm AS' prediction of rational cooperation.
- Point estimates $\hat{\bar{\tau}}^{\dagger}(1,0)(p) > 0$ for $p > 0 \implies$ suggestive evidence of dynamic causal effects.
    - Treatment may induce participants to learn value of cooperation, thereby producing persistent effects.
- **In the paper**: additionally report results for period-specific and unit-specific weighted average dynamic causal effects.

Conclusion

- Develop potential outcome model for studying dynamic causal effects in panel experiments.
- **Key Ideas + Results**:
  1. Define new panel-based dynamic causal estimands and introduce associated nonparametric estimator.
  2. Show non-parametric estimator is unbiased over randomization dist and derive novel finite population CLTs.
  3. Derive finite population probability limit of linear fixed effects estimators.
- Potential outcome panel is a rich framework for exploring dynamic causal effects in panel data.

Thank you!

asheshr@g.harvard.edu

Appendix

Horvitz-Thompson estimator back

- **Theorem 3.1**: Under individualistic + probabilistic assignment,

$$\mathbb{E}[\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) \mid \mathcal{F}_{i,t-p-1}] = \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p),$$
$$Var(\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) | \mathcal{F}_{i,t-p-1}) = \gamma^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}) - \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)^2,$$

where

$$\gamma^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) = \frac{Y_{i,t}(w^{obs}_{i,1:t-p-1}, \mathbf{w})^2}{p_{i,t-p}(\mathbf{w})} + \frac{Y_{i,t}(w^{obs}_{i,1:t-p-1}, \tilde{\mathbf{w}})^2}{p_{i,t-p}(\tilde{\mathbf{w}})}.$$

Further, for distinct $\mathbf{w}, \tilde{\mathbf{w}}, \bar{\mathbf{w}}, \hat{\mathbf{w}} \in \mathcal{W}^{(p+1)}$

$$Cov(\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p), \hat{\tau}_{i,t}(\bar{\mathbf{w}}, \hat{\mathbf{w}}; p) | \mathcal{F}_{i,t-p-1}) = -\tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)\tau_{i,t}(\bar{\mathbf{w}}, \hat{\mathbf{w}}; p).$$

Finally, $\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}})$ and $\hat{\tau}_{j,t}(\mathbf{w}, \tilde{\mathbf{w}})$ are independent for $i \neq j$ conditional on $\mathcal{F}_{1:N,t-p-1}$.

Variance estimation for Horvitz-Thompson estimator back

- Variance of $\hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)$ depends upon the potential outcomes under both the treatment and counterfactual and is generally not estimable.

- However, it is bounded from above by $\gamma^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)$, which we can estimate by

$$\hat{\gamma}^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) = \frac{(y^{obs}_{i,t})^2 \{\mathbb{1}(w^{obs}_{i,t-p:t} = \mathbf{w}) + \mathbb{1}(w^{obs}_{i,t-p:t} = \tilde{\mathbf{w}})\}}{p_{i,t-p}(w^{obs}_{i,t-p:t})^2}.$$

- **Lemma 3.2**: Under set-up of Theorem 3.1, $\mathbb{E}[\hat{\gamma}^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p) | \mathcal{F}_{i,t-p-1}] = \gamma^2_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)$.

Finite population CLT and conservative inference back

- Estimate unit and time averaged dynamic causal effects by averaging the HT estimator.
- Example: the estimator of the time-t, lag-p average dynamic causal effect is

$$\hat{\bar{\tau}}_{\cdot t}(\mathbf{w}, \tilde{\mathbf{w}}; p) := \frac{1}{N}\sum_{i=1}^{N} \hat{\tau}_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)$$

and its variance is

$$\sigma_{\cdot t}^2 := \frac{1}{N}\sum_{i=1}^{N}\{\gamma_{i,t}^2(\mathbf{w}, \tilde{\mathbf{w}}; p) - \tau_{i,t}(\mathbf{w}, \tilde{\mathbf{w}}; p)^2\}.$$

Finite population CLT and conservative inference back

- **Theorem 3.2**: Under individualistic + probabilistic assignment and bounded potential outcomes

$$\frac{\sqrt{N}\{\hat{\bar{\tau}}_{\cdot t}(\mathbf{w},\tilde{\mathbf{w}};p)-\bar{\tau}_{\cdot t}(\mathbf{w},\tilde{\mathbf{w}};p)\}}{\sigma_{\cdot t}} \xrightarrow{d} N(0,1) \quad \text{as } N\to\infty,$$
$$\frac{\sqrt{T-p}\{\hat{\bar{\tau}}_{i\cdot}(\mathbf{w},\tilde{\mathbf{w}};p)-\bar{\tau}_{i\cdot}(\mathbf{w},\tilde{\mathbf{w}};p)\}}{\sigma_{i\cdot}} \xrightarrow{d} N(0,1) \quad \text{as } T\to\infty,$$
$$\frac{\sqrt{N(T-p)}\{\hat{\bar{\tau}}(\mathbf{w},\tilde{\mathbf{w}};p)-\bar{\tau}(\mathbf{w},\tilde{\mathbf{w}};p)\}}{\sigma} \xrightarrow{d} N(0,1) \quad \text{as } NT\to\infty.$$

Autoregressive potential outcome panel back

- Autoregressive potential outcome panel satisfies $Y_{i,1}(w_{i,1}) = \beta^*_{i,1,0} w_{i,1} + \epsilon_{i,1}$ and, for all $t > 1$,

  $$Y_{i,t}(w_{i,1:t}) = \phi_{i,t,0} Y_{i,t-1}(w_{i,1:t-1}) + \ldots + \phi_{i,t,t-2} Y_{i,1}(w_{i,1}) + \beta^*_{i,t,0} w_{i,t} + \ldots + \beta^*_{i,t,t-1} w_{i,1} + \epsilon^*_{i,t}$$

  Coefficients $\phi_{i,t,0:t-2}, \beta^*_{i,t,0:t-1}$ and residuals $\epsilon^*_{i,1:t}$ do not depend on treatments.

- Allows for arbitrary heterogeneity in the parameters across units and arbitrary dependence across units and time through $\epsilon^*_{i,t}$.

Estimation with linear models: unit fixed effects back

- Let the within-unit transformed variable be $\check{A}_{i,t} = A_{i,t} - \bar{A}_{i\cdot}$.
  Denote $Cov(\check{W}_{i,t}, \check{W}_{i,s}) = \check{\sigma}_{W,i,t,s}$ and $\check{\mu}_{i,t} = \mathbb{E}\left[\check{W}_{i,t} | \mathcal{F}_{1:N,0,T}\right]$.

- **Proposition 4.2**: Assume the potential outcome panel is linear, the assignment mechanism is individualistic and $Var(\check{W}_{i,t} | \mathcal{F}_{1:N,0,T}) = \check{\sigma}^2_{W,i,t} < \infty$ for each $i \in [N], t \in [T]$. Further assume that as $N \to \infty$, the following sequences converge non-stochastically:

$$N^{-1} \sum_{i=1}^{N} \beta_{i,t,s} \check{\sigma}_{W,i,t,s} \to \check{\kappa}_{W,\beta,t,s},$$

$$N^{-1} \sum_{i=1}^{N} \check{\sigma}^2_{W,i,t} \to \check{\sigma}^2_{W,t}, \quad N^{-1} \sum_{i=1}^{N} \check{Y}_{i,t}(\mathbf{0}) \check{\mu}_{i,t} \to \check{\delta}_t.$$

  Then, as $N \to \infty$,

$$\hat{\beta}_{UFE} \xrightarrow{p} \frac{\sum_{t=1}^{T} \check{\kappa}_{W,\beta,t,t}}{\sum_{t=1}^{T} \check{\sigma}^2_{W,t}} + \frac{\sum_{t=1}^{T} \sum_{s=1}^{t-1} \check{\kappa}_{W,\beta,t,s}}{\sum_{t=1}^{T} \check{\sigma}^2_{W,t}} + \frac{\sum_{t=1}^{T} \check{\delta}_t}{\sum_{t=1}^{T} \check{\sigma}^2_{W,t}}.$$

Estimation w/ linear models: unit fixed effects, example back

- Consider linear potential outcome model w/ lag-1 causal effects

$$Y_{i,t}(w_{i,1:t}) = \beta_0 w_{i,t} + \beta_1 w_{i,t-1} + \epsilon_{i,t}$$

  and $Y_{i,1}(w_{i,1}) = \beta_0 w_{i,1} + \epsilon_{i,1}$ for $t = 1$.

- Proposition 4.2 establishes

$$\hat{\beta}_{UFE} = \underbrace{\beta_0}_{(1)} + \underbrace{\beta_1 \frac{\sum_{t=2}^{T} \sigma_{\check{W},t,t-1}}{\sum_{t=1}^{T} \sigma^2_{\check{W},t}}}_{(2)} + \underbrace{\frac{\sum_{t=1}^{T} \check{\delta}_t}{\sum_{t=1}^{T} \sigma^2_{\check{W},t}}}_{(3)},$$

  where (1) contemporaneous dynamic causal coefficient, (2) depends on lag-1 dynamic causal coefficient and autocovariance b/w assignments across periods.

Andreoni & Samuelson (2006): design of panel experiment back

- Two-period prisoners' dilemma stage game:
  - Payoffs across two periods determined by $x_1, x_2 \geq 0$ such that $x_1 + x_2 = 10$.
  - Define $\lambda = \frac{x_2}{x_1+x_2}$, which govern rel. payoffs b/w two periods.
    $\lambda = 0 \rightarrow$ all payoffs occurred in period one.
    $\lambda = 1 \rightarrow$ all payoffs occurred in period two.

  Predict larger λ, players will cooperate more often in period one.

	C	D
C	$(3x_1, 3x_1)$	$(0, 4x_1)$
D	$(4x_1, 0)$	(x_1, x_1)

Period one

	C	D
C	$(3x_2, 3x_2)$	$(0, 4x_2)$
D	$(4x_2, 0)$	(x_2, x_2)

Period two

Andreoni & Samuelson (2006): design of panel experiment back

- Panel experiment conducted over 5 sessions.
- In each session, 22 subjects were recruited to play 20 games of the twice-played prisoners' dilemma.
  - In each game, participants randomly matched into pairs.
  - Each pair then randomly assigned λ from set $\{0, 0.1, \ldots, 0.9, 1\}$.
- Authors analyze experimental data using regression models w/ unit-level fixed effects.

Rational cooperation in prisoners' dilemma back