

# An Economic Approach to Regulating Algorithms

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# Motivation

- Algorithms are increasingly used in a wide variety of important domains.
- **Criminal justice:** Should a defendant be granted bail?
- **Medicine:** Should a patient be tested?
- **Hiring:** Should an applicant be hired?
- **Finance:** Should an applicant receive a loan?
- Widespread fears that algorithmic decision-making may **reflect** or **worsen** existing socioeconomic disparities.

# Algorithmic Fairness: Perspective from Computer Science

- An algorithm produces a **prediction function**  $\hat{f}$ , which predicts label  $Y^*$  from features  $W$ .
  - ⇒ Define what it means for  $\hat{f}$  to be **fair**.
  - ⇒ Take chosen def'n of “fair prediction function” as **primitive**.
- Constructing fair algorithms reduces to introducing an **additional constraint** in our training procedures:

$$\min_f \frac{1}{N} \sum_{i=1}^N \mathcal{L}(Y_i^*, W_i; f) \text{ s.t. } f \text{ is “fair.”}$$

- Enormously influential literature: Dwork et al. (2012), Zemel et al. (2013), Hardt et al. (2016) and many, many others.

# This Paper: An Economic Approach

- Embed concerns about algorithmic bias within a **social welfare function**.
  - ▶ Defined over outcomes generated by decisions.
  - ▶ Captures explicit preference for **efficiency** and **equity**.
- Explicit equity preference in SWF generates concern about possible algorithmic bias.
- **Our Approach**: begin w/ SWF and **derive implications** of equity preferences for algorithm construction.
  - ▶ Analogy to optimal tax: derive properties of tax system, taking SWF as primitive.

# This Paper: An Economic Approach

- **Our Approach:** Cast questions surrounding design/use of algorithms and algorithmic fairness into canonical welfare economics framework.
- Welfare economics framework highlights two distinct **problem formulations**.
- **First-Best Problem:** Benevolent “social planner” has full control over design/use of algorithmic decision rule.
  - Statistical decision theory
- **Second-Best Problem:** Third-party DMs control design/use of algorithmic decision rule, and do not share same objectives as society.
  - What are DMs maximizing? What information do they have? What information does the social planner have about DMs? What policy tools are available?
  - Contracting problem, Mechanism design

# This Paper: An Economic Approach

- **Our Approach:** Cast questions surrounding design/use of algorithms and algorithmic fairness into canonical welfare economics framework.
- **Big Picture Goal:** Provide framework for economists to think about algorithmic fairness, and thereby fruitfully collaborate w/ computer science community.
  - ▶ Richer analyses of second-best/regulation problem requires tractable modelling of both *statistical* and *economic* aspects of the problem.

# Screening decisions

- **Screening Decisions:** Individuals screened into program based on prediction of unknown outcome of interest  $Y^* \in \{0, 1\}$ 
  - ▶ Common example of “*prediction policy problem*” (Kleinberg et al., 2015, 2018a)
- **Examples** include pre-trial release, hiring decisions, credit approvals and more.
- Population of individuals described by characteristics  $W \in \{0, 1\}^J$  and group membership  $G \in \{0, 1\}$ .
  - ▶  $G = 1$  for “*protected group*.”
  - ▶  $P(g, w) = \mathbb{P}\{G = g, W = w\}$ ,  $\theta^*(g, w) = \mathbb{E}[Y^* \mid G = g, W = w]$ .
- Predict  $Y^*$  given observed features  $(G, W)$  and admit individuals based upon predictions.

# What algorithm would social planner use?

- Consider social planner that constructs algorithm and selects admissions rule in the screening decision.
  - ▶ Equivalently: Benevolent private actor that shares society's goals.
- This is the **first-best problem**. Analyze first-best to understand how equity preferences affect screening decisions.
- **Questions**: What algorithm would social planner construct? What admissions rule would social planner select?



# The social welfare function

- Weighted average of expected outcome of interest among admitted individuals:

$$\sum_{g,w} \psi_g \theta^*(g, w) t(g, w) P(g, w),$$

- ▶ Admissions rule  $t(g, w) \in [0, 1]$
  - ▶ Generalized social welfare weights,  $\psi_g \geq 0$ .
- SWF captures preference for both **efficiency** and **equity**
  - ▶ Efficiency: maximize expected outcome of interest among admitted group
  - ▶ Equity: value outcomes associated w/ protected group more,  $\psi_1 > \psi_0$ .
- Social planner does not know  $\theta^*(g, w) \implies$  faces non-trivial *prediction policy problem*.

# Measured outcomes and the training dataset

- Social planner receives access to **training dataset** to construct predictions.
  - ▶ Training dataset  $D_N$  consists of  $N$  i.i.d. random draws from population.
- Each individual additionally described by **measured outcome**  $\tilde{Y}$ .
  - ▶ May differ from true outcome of interest  $Y^*$ .
- Training dataset useful provided measured outcome  $\tilde{Y}$  informative about outcome of interest  $Y^*$ .
  - ▶ Specifies **prior beliefs**  $\pi(\cdot)$  over  $(Y^*, \tilde{Y})|G, W$ .
  - ▶ Knows marginal dist  $(G, W)$ .

# The first-best screening problem

- Social planner faces known **capacity constraint**  $C \in [0, 1]$ . May not admit more than fraction  $C$  of population into program.
- **First-best problem**: Maximize expected social welfare subject to capacity constraint

$$\begin{aligned} \max_{t(g,w;D_N)} \mathbb{E}_\pi & \left[ \sum_{(g,w)} \psi_g \theta^*(g,w) t(g,w;D_N) P(g,w) \right] \\ \text{s.t.} \quad & \sum_{(g,w)} t(g,w;D_N) P(g,w) \leq C \quad \text{w/ probability one.} \end{aligned}$$

Solution is **first-best admissions rule**.

# First-best admissions rule

- **Proposition:** First-best admissions rule is *threshold rule* w/ group-specific admissions thresholds

$$1 \{ \mathbb{E}_{\pi|D_N} [Y^* | G = g, W = w] > \tau^*(g; C) \},$$

w/ ties handled s.t. capacity constraint binds.

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- Social planner uses all available info in training data + prior to construct rank-ordering of population.
  - ▶ *Intuition:* Provided data informative about outcome, social planner “uses” data to update prior beliefs. [Details](#)
  - ▶ *Intuition:* Construct optimal prediction of measured outcome  $\check{Y}$ . Use prior beliefs  $\pi$  to map into predictions of outcome of interest  $Y^*$ . [Details](#)

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- **Takeaways:** Deliver best estimate of  $\mathbb{E}[\tilde{Y} | W, G]$ .

- 1 Do not blind algorithm to group membership  $G$ .
- 2 Do not remove any characteristics  $W$ .
- 3 Do not place additional constraints in training procedure.

Equity preferences modify admissions rule, not prediction function.

# First-best admissions rule

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w/ ties handled s.t. capacity constraint binds.

- **Empirical Question:** Is it really reasonable to assume social planner able to specify full prior over conditional joint distribution  $(Y^*, \tilde{Y}) | W, X$ ?
- **Modelling Question:** Model assumes social planner is fully Bayesian; what if instead she is ambiguity-averse (i.e.,  $\Gamma$ -minimax)?
  - Are there decision-theoretic justifications for fairness criteria?

# How would social planner regulate decision-makers?

- **In many applications:** third-party firms control construction of prediction function and admissions rule
  - ▶ Examples: resume screening, credit approvals.Some firms may wish to discriminate against protected group.
- This is a **regulation problem**.
  - ▶ Social planner interacts w/ third-party DM. Takes their screening decisions as given.
  - ▶ Limited policy instruments to influence DM's choices.
- **Next:** extend model to analyze second-best problem.



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- **Bigger Picture:** Exciting area w/ room for fruitful collaborations b/w economics and computer science.
  - How to tractably incorporate *both* richer statistics *and* richer economics into the model?

# How would social planner regulate decision-makers?

- Social planner oversees market of human DMs and each faces own screening decision.
- Two constraints:
  - ① **Policy constraint:** May only enforce *model regulations* – restrict which  $W$  can be used in admissions rules.
  - ② **Information constraint:** Does not know which human DMs are discriminatory and knows less about which  $W$  have signal for predicting  $Y^*$ .
- Two sets of results:
  - ① Model captures existing intuitions about regulating discrimination – e.g., disparate treatment and disparate impact.
  - ② Analyze how algorithmic decision-making changes this regulation problem.

# Market of human decision-makers

- Human DM's **preferences**  $\lambda = (\lambda_0, \lambda_1)$  governs payoffs

$$U(t; \lambda) = \sum_{(g,w)} \lambda_g \theta^*(g, w) t(g, w) P(g, w),$$

Specifies relative weights placed on outcomes associated with each group.

- Only two types of preferences in the market:
  - ① **Non-discriminatory** preferences w/  $\lambda_0 = \lambda_1 = 1$ ,
  - ② **Discriminatory** preferences w/  $\bar{\lambda}_0 = 1 > \bar{\lambda}_1$
- Discriminatory firms are **taste-based discriminators** in the spirit of Becker (1957).
  - ▶ How to incorporate richer models of discriminatory behavior? E.g., stereotypes as in Bordalo et al. (2016)

# Market of human decision-makers

- Human DMs **prior beliefs**  $\pi_m$  describe beliefs about which  $W$  are relevant in predicting  $Y^*$ .
- Each prior  $\pi_m$  assoc. w/ model  $m \subseteq \{1, \dots, J\}$ .
  - ▶ Only characteristics in model  $m$  contain signal for predicting outcome of interest  $Y^*$ .
- Assume each prior  $\pi_m$  additionally satisfies:
  - 1 **Sufficiency**: No average group differences in  $Y^*$  conditional on characteristics in model  $m$ .
  - 2 **Relevance**: All characteristics in model  $m$  contain signal on average for  $Y^*$ .

Details

# Market of human decision-makers

- Each human DM faces **capacity constraint**  $C \in [0, 1]$ . May not admit more than fraction  $C$  of population into program
- Market of human DMs characterized by joint dist'n  $\eta(\lambda, \pi_m, C)$ .
  - ▶ Full support and assume capacity constraint is independent of preferences and beliefs under  $\eta$ .
- Selects admissions rule to maximize expected payoffs subject to capacity constraint. Optimal admissions rule is **threshold rule**.
  - ▶ Thresholds depend on preferences  $\lambda$ .

# The social welfare function

- For single screening problem, SWF defined as before.
- Assume preferences are **aligned** w/ non-discriminatory human DMs.
  - ▶ Social planner's preferred rank-ordering is same non-discriminator's preferred rank-ordering w/  $(\psi_0, \psi_1) = (1, 1)$ .
- Social planner only knows joint dist'n  $\eta$  of  $(\lambda, \pi_m, C)$ . Payoffs across market summarized by **aggregate social welfare function**

$$\int_C \left( \sum_{(g,w)} \mathbb{E}_\eta [\theta_{\pi_m}^*(w) t(g, w)] P(g, w) \right) h(C) dC.$$

# Model regulations

- Only policy instrument available is **model regulations**. [Details](#)
  - ▶ Social planner may regulate what characteristics can be used in admissions rules.
- Banning characteristics  $\implies$  select rank-ordering more closely matches social planner's preferred rank-ordering.
  - ▶ At model controls  $m$ , must pool across all characteristics outside of model  $m$ .
  - ▶ If group membership banned, further pool across groups.

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  - ▶ If group membership banned, further pool across groups.
- Are model regulations actually enforceable?
  - ▶ *Interpretation*: admissions rules are directly observable.
  - ▶ *Counterpoint*: For human decision-makers, admissions rules are mechanically **unobservable!**



# Model regulation in practice

- Social planner can enforce model regulations  $\implies$  social planner directly observes admissions rules  $t(g, w)$
- **In practice:** social planner faces additional inference problem in regulating discrimination.
  - ▶ Must infer admissions rules from finitely many admissions decisions.

# Model regulation in practice

- Social planner can enforce model regulations  $\implies$  social planner directly observes admissions rules  $t(g, w)$
- Inferring whether admissions rule uses group membership  $\iff$  testing whether admissions decisions are conditionally independent of group membership.
- Conditional independence testing is a **hard** statistical problem.
  - ▶ Shah and Peters (2019): no uniformly valid test in general.
  - ▶ For discrete distributions, hardness grows w/ dimensionality.  
Canonne et al. (2018): Tests with good size/power properties require number of samples to grow exponentially in dimensionality of observable characteristics.
- How to incorporate statistical problems into framework of optimal model regulation?

# The social planner's second-best problem

- **Second-best problem:** Select model regulations to max aggregate social welfare, taking admissions rules chosen by human DMs as given

$$\arg \max_{m \subseteq \{1, \dots, J\}} \int_C \left( \sum_{(g, w)} \psi_g \mathbb{E}_\eta [\theta_{\pi_{\tilde{m}}}^*(w) t_{\lambda, C}^{\tilde{m}}(g, w; m) P(g, w)] \right) h(C) dC.$$

- Social planner searches over model controls to find regulations that induce rank-ordering most closely aligned w/ social planner's first-best rank-ordering.
- Optimal model regulations may be quite complex.
  - ▶ Depends on fractions of discriminators vs. non-discriminators as well as dist'n of beliefs  $\pi_m$

# Flexibility tradeoff in model regulations

- Non-discriminators only use characteristics in admissions rule if they are predictive of the outcome  $Y^*$
- Discriminators additionally may use characteristics to screen out disadvantaged group.
- **Flexibility tradeoff:** Letting human DMs use additional characteristic leads to two effects
  - ① Improves rank-orderings of population
  - ② Used by some discriminators to screen out disadvantaged group
- Optimal model regulations involve **disparate impact tests**.
  - ▶ Does this variable provide sufficient predictive power for  $Y^*$  across market relative to its predictive power of group membership  $G$ ?

# Algorithmic Decision-Making and Second-Best Model Regulations

- Considered social planner's second-best problem when faced w/ market of human DMs.
- **Now:** introduce **algorithmic decision-making**. How does it change second-best model regulations?
  - ▶ Model algorithms as revealing *ground truth*  $\theta^*(g, w)$  in each screening problem.
  - ▶ *Ground-truth model*  $m$
  - ▶ *Assumption:* Firms cannot manipulate ground-truth model.
- Depends on **disclosure regime**: what must human DMs disclose about their algorithms?
  - 1 Only admissions rules  $t(g, w)$ .
  - 2 Both admissions rule and all model inputs (training data, training procedures, etc.).

## Second-best model regulations with known admissions rules

- **First disclosure regime:** only disclose admissions rule to social planner.
- **Result:** Nothing fundamentally changes! Social planner still faces flexibility tradeoff.
  - ▶ *Intuition:* social planner still faces asymmetric information over both ground-truth model and preferences.
- **In practice:** Algorithmic decision-making forces human DMs to specify an admissions rule.
  - ▶ No longer need to infer admissions rule from finitely many admissions decisions (Kleinberg et al., 2018b).
  - ▶ Optimal regulation of admissions rules is now *feasible*.

## Second-best model regulations with algorithmic audits

- Algorithmic decision-making introduces new policy tool – **algorithmic audits**
  - ▶ Refers to access of underlying training data and training procedures used to construct algorithm (Kleinberg et al., 2018b).
- Algorithmic audits reveal ground-truth model  $\theta_m^*$  of each human decision-maker.
  - ▶ Eliminate one dimension of private information.
- Social planner may condition model regulations on ground-truth model.

$$\arg \max_{\tilde{m} \subseteq \{1, \dots, J\}} \int_C \left( \sum_{(g, w)} \psi_g \mathbb{E}_{\lambda|m} [\theta_m^*(w) t_{\lambda, C}^m(g, w; \tilde{m}) P(g, w)] \right) h(C) dC.$$

**Algorithmic second-best problem.**

## Second-best model regulations with algorithmic audits

- **Proposition:** Social planner allows human DMs to use any characteristics predictive of  $Y^*$  at revealed ground-truth model.
- Social planner knows ground-truth model  $\implies$  understand *why* a characteristic is included in admissions rule.
  - ▶ If characteristic used in admissions rule but not predictive of  $Y^*$ , then must be to screen out disadvantaged group!
- Requires presence of **algorithmic audits**.



# Conclusion

- **This paper:** Developed economic model of screening decisions and embedding concerns about algorithmic bias within a social welfare function.
- **First-best problem:** social planner constructs prediction function and selects admissions rule.
  - ▶ *Equity irrelevance result:* equity preferences alter admissions rule, not prediction function.
- **Second-best/regulation problem:** possibly discriminatory human DMs construct prediction function and admissions rule.
  - ▶ *Algorithmic audits:* optimal to let human DMs use any characteristics that are predictive of the outcome of interest.

# Conclusion

- **This paper:** Developed economic model of screening decisions and embedding concerns about algorithmic bias within a social welfare function.
- Optimal regulation of algorithmic decision rules is a ripe area for research and policy.
  - ▶ Analysis of the algorithmic regulation problem is area for fruitful collaboration b/w economics and computer science.
  - ▶ Several dimensions along which to enrich both the *statistics* and the *economics* of the model.

# Interpreting the first-best admissions rule

- Training data **ignored** if

$$\mathbb{E}_{\pi|D_N}\theta^*(g, w) = \mathbb{E}_{\pi}\theta^*(g, w)$$

for all  $(g, w) \in \{0, 1\}^{J+1}$  and training datasets  $D_n$  that occur w/ pos. probability.

- **Proposition:** social planner ignores  $D_N$  iff “ $Y^* \perp\!\!\!\perp \tilde{Y} \mid W, G$ ” under her prior beliefs  $\pi$ .
  - ▶ Application of results in Poirier (1998) on Bayesian inference in partially identified models.
- If  $\tilde{Y}$  related to  $Y^*$  in **any way** under prior beliefs, then  $D_N$  is useful in screening decisions!
  - ▶  $\tilde{Y}$  is mis-measured, negatively correlated w/  $Y^*$ , positively correlated w/  $Y^*$  or biased against protected group!

# Interpreting the first-best admissions rule

- **Another interpretation:** Construct optimal prediction of measured outcome  $\tilde{Y}$  and then use prior beliefs  $\pi$  to map into predictions of outcome of interest  $Y^*$ .
  - ▶ Extend results in Moon and Schorfheide (2012) to formalize statement.
- **Notation:** Let  $\pi(\theta^*|\tilde{\theta})$  denote social planner's conditional beliefs about  $\theta^*$  given  $\tilde{\theta}$ .
- **Proposition:** Suppose  $\hat{f}_N$  is a consistent prediction function for  $\mathbb{E}[\tilde{Y} | G, W]$ .  
Social planner's plug-in posterior beliefs  $\pi(\theta^*|\hat{f}_N)$  asymp. approx. social planner's true posterior beliefs  $\pi(\theta^*|D_N)$

$$d_{TV} \left( \pi(\theta^*|D_n), \pi(\theta^*|\hat{f}_N) \right) \xrightarrow{P} 0 \text{ as } N \rightarrow \infty.$$

## Human decision-makers: prior beliefs

- **Formal:** Joint dist'n over parameters  $\{\theta^*(g, w) : (g, w) \in \{0, 1\}^{J+1}\}$  satisfying

$$\mathbb{E}_{\pi_m} [\theta^*(g, w_m, w_{-m})] = \mathbb{E}_{\pi_m} [\theta^*(g, w_m, w'_{-m})]$$

for all  $g \in \{0, 1\}$ ,  $w_m \in \{0, 1\}^{|m|}$ ,  $w_{-m}, w'_{-m} \in \{0, 1\}^{J-|m|}$ .

- **Sufficiency:** At  $m \subseteq \{1, \dots, J\}$  and associated beliefs  $\pi_m$ ,

$$\theta_{\pi_m}^*(0, w_m) = \theta_{\pi_m}^*(1, w_m) \quad \forall w_m \in \{0, 1\}^{|m|}.$$

- **Relevance:** At  $m \subseteq \{1, \dots, J\}$  and associated beliefs  $\pi_m$ ,

$$\theta_{\pi_m}^*(g, w_m) \neq \theta_{\pi_m}^*(g, w'_m) \quad w_m, w'_m \in \{0, 1\}^{|m|} \quad w_m \neq w'_m.$$

# Model regulations

- **Model regulations:** Social planner may regulate what characteristics can be used in the human DMs' admissions rules
- If social planner implements **model regulations**  $m$ , then all admissions rules must satisfy

$$t(g, w_m, w_{-m}) = t(g, w_m, w'_{-m})$$

for all  $g \in \{0, 1\}$ ,  $w_m \in \{0, 1\}^{|m|}$  and  $w_{-m}, w'_{-m} \in \{0, 1\}^{J-|m|}$ .

- If group membership banned, then admissions rules must satisfy  $t(g, w_m, w_{-m}) = t(g', w_m, w'_{-m})$ , for all  $g, g'$ .

Back

## Disadvantage condition

- **Disadvantage condition:** characteristics associated w/ lower avg. values of  $Y^*$  more likely to occur among protected group.
- At each beliefs  $\pi_m$ , if  $w, w'$  are s.t.  $\theta_{\pi_m}^*(w) \geq \theta_{\pi_m}^*(w')$ , then

$$\frac{P(0, w)}{P(1, w)} \geq \frac{P(0, w')}{P(1, w')}.$$

Holds w/ strict inequality if  $\theta_{\pi_m}^*(w) > \theta_{\pi_m}^*(w')$ .

- Equivalent to dist'n for protected group is *likelihood ratio dominated* by dist'n of rest of population.

Back

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